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Determination of free surface and gravitational flow of liquid in triangular groove

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The problem of determination of friction factor (or average fluid velocity) for a steady gravitational flow of fluid in an inclined open triangular groove is addressed. It is assumed that the angle of slope of the groove is known and the gravity is the only driving force of flow.

The solution comprises of two parts. First, for a given groove's basal angle, liquid-solid contact angle, and the Bond number, the shape of free surface is determined starting from the Young-Laplace equation. The optimization technique as developed formerly is applied. Then, having determined the shape of the free surface and slope of the groove, the distribution of fluid flow is determined. The boundary value problem is solved using boundary collocation method in least square sense. Given the distribution of fluid velocity the friction factor as a function of the other parameters of the model is analyzed.

1 Introduction

The problem of laminar flow of viscous incompressible liquid in triangular grooves due to gravity forces and in presence of surface tension are considered in connection with such processes as: evaporation of water from saline water (Ayyaswamy, Catton and Edwards 1974) or other processes of intensive evaporation (Ha and Peterson 1994), metal casting technology, and modeling of capillary flow in porous media (Romero and Yost 1996). A theoretical description of the problem was elaborated in the paper (Ayyaswamy, Catton and Edwards 1974) assuming that the free surface of liquid has a constant radius of curvature, which corresponds to a very low Bond number. Although such assumption is convenient from the point of view of a numerical solution of the problem giving an analytical form of the boundary condition pertaining to the free surface, it constitutes the approximate condition which may not be sufficient to obtain the exact prediction of the shape of the free surface as well as the intensity of evaporation and flow. The authors paper (Kaczmarek, Kolodziej and Musielak 1988) is focused on a method of numerical determination of the shape of free surface in triangular groove assuming variable amount of liquid, and wide ranges of coefficients of surface tension and density of liquid (i.e. for wide distribution of the Bond number) assuming that there is no flow of liquid in the groove.

The purpose of the present paper is to develop a numerical method allowing for determination of laminar flow of viscous incompressible liquid in triangular groove due to gravity force taking into account variable and initially unknown curvature of the free surface (interfacial tension). The numerical data obtained concern a wide range of the Bond number. The paper includes formulation of the mathematical problem (Sect. 2), followed by the description of the proposed solution procedure (Sect. 3), and analysis of the numerical results (Sect. 4).

2 Formulation of the problem

The stationary flow of liquid in a triangular groove (V-shaped groove) at an inclination angle β is considered. The driving force of the flow is gravity force on liquid which has the specific weight equal to γ. It is assumed that the shape of the free surface of liquid is essentially influenced by the interfacial tension. This means that the dimensionless number which characterizes the surface tension, the Bond number $\mathrm{Bo} = \gamma b^2/\sigma$, assumes small values, where σ is the surface tension coefficient, b denotes the distance from the free surface to the bottom of the groove when there is no capillary forces. The basal angle of the groove is 2Φ and the liquid contact angle amounts Θ.

A cylindrical coordinate system, $(r,\ \varphi,\ z)$, with its z axis coincident with the edge of the groove and $\varphi = 0$ for symmetry axis is chosen (Fig. 1).

A kind of superposition assumption is accepted which allows us to separate the problem of determination of the shape of free surface from the problem of flow of liquid, while the reverse procedure is not possible. It means that the shape of the free surface is assumed to be identical with that for the case of stagnant liquid (is not influenced by the flow) and the shape of free surface influences the picture of the fluid flow.

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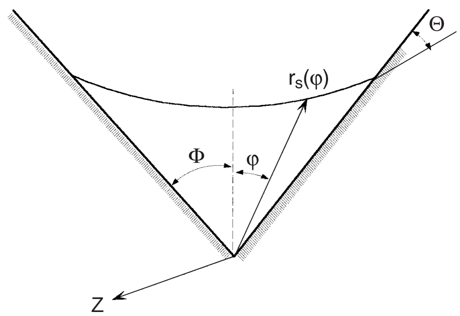

Fig. 1. Groove geometry and co-ordinate system

The geometry of free surface is described by the following Young-Laplace relation:

$$\Delta p = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{1}$$

where Δp denotes the pressure difference across an interface, R_1 and R_2 are the local principal radii of curvature of the free surface. Since the considered free surface has a cylindrical shape we assume that $R_2 = \infty$.

In what follows it is assumed that the pressure in any crossection of the liquid in triangular groove is governed by the principles of steady state behavior of fluid. The latter assumption is valid for laminar flow and moderate inclination angles. Thus, the pressures above and below the free surface, p_1 and p_2, respectively are:

$$p_1 = -\gamma_1(r\cos\varphi - C_1) \tag{2}$$

$$p_2 = -\gamma_2(r\cos\varphi - C_2) \tag{3}$$

where γ_1 and γ_2 denotes specific weights of fluid above and below the free surface, and C_1 and C_2 are integration constants. The jump of pressure across the interface is then given as follows

$$\Delta p = p_1 - p_2 = \gamma(r_s\cos\varphi - C) \tag{4}$$

where $\gamma = \gamma_2 - \gamma_1$ and for the case $\gamma_1 \ll \gamma_2$ (liquid-vapor interface) it is assumed that $\gamma = \gamma_2$. The constant $C = C_1 - C_2$ determines the height of liquid for the case when $p_2 = p_1$.

Combining relation (4) and the equation which defines the radius of curvature of free surface in cylindrical co-ordinates the relation (1) can be written as follows:

$$\gamma(r_s\cos\varphi - C) = -\sigma\frac{r_s^2 + 2\left(\frac{\mathrm{d}r_s}{\mathrm{d}\varphi}\right)^2 - r_s\frac{\mathrm{d}^2 r_s}{\mathrm{d}\varphi^2}}{\left[r_s^2 + \left(\frac{\mathrm{d}r_s}{\mathrm{d}\varphi}\right)^2\right]^{3/2}} \ . \tag{5}$$

The symmetry of the problem allows one to set the following boundary condition for the center of the free surface:

$$\frac{\mathrm{d}r_s(\varphi)}{\mathrm{d}\varphi} = 0 \quad \text{for} \quad \varphi = 0 \ . \tag{6}$$

In turn the boundary condition at the edge of the free surface is determined by the angle at the contact of the three media (liquid, vapor, and solid) and can be written as following:

$$\frac{\mathrm{d}r_s(\varphi)}{\mathrm{d}\varphi} = \frac{r_s}{\tan\theta} \quad \text{for} \quad \varphi = \Phi \ , \tag{7}$$

where the contact angle is a constant given in physical tables.

An additional condition which must be satisfied by the solution of Eq. (5) results from the assumed amount of liquid in the groove and reads:

$$\int_0^\Phi r_s^2(\varphi)\mathrm{d}\varphi = b^2\tan\Phi \ . \tag{8}$$

The condition (8) may be used to determine the unknown integration constant C.

Introducing a new dimensionless variable $R = r_s/b$ the Eq. (5) and boundary conditions (6–8) can be rewritten as the following two point boundary value problem:

$$\frac{\mathrm{d}^2 R_s}{\mathrm{d}\varphi^2} = R_s + \frac{2}{R_s}\left(\frac{\mathrm{d}R_s}{\mathrm{d}\varphi}\right)^2 + \mathrm{Bo}\left(\cos\varphi - \frac{A}{R_s}\right)\left[R^2 + \left(\frac{\mathrm{d}R_s}{\mathrm{d}\varphi}\right)^2\right]^{\frac{3}{2}} \tag{9}$$

$$\frac{\mathrm{d}R_s}{\mathrm{d}\varphi} = 0 \quad \text{for} \quad \varphi = 0 \tag{10}$$

$$\frac{\mathrm{d}R_s}{\mathrm{d}\varphi} = \frac{R}{\tan\Theta} \quad \text{for} \quad \varphi = \Phi \tag{11}$$

$$\int_0^\Phi R_s^2(\varphi)\mathrm{d}\varphi = \tan\Phi \ , \tag{12}$$

where $A = C/b$ is a dimensionless constant.

The problem of two-dimensional laminar flow of incompressible viscous liquid (the Poiseulle type of flow) in grooves is governed by the two-dimensional Poisson's equation with the right hand side to be a constant:

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\varphi^2} = -\frac{1}{\mu}\gamma\sin\beta \ , \tag{13}$$

where $w(r,\varphi)$ is the flow velocity in direction perpendicular to the edge of groove, and μ denotes the dynamic viscosity of liquid.

If one introduces a dimensionless velocity

$$W = \frac{w}{(b^2\gamma\sin\beta)/\mu} \tag{14}$$

Eq. (13) reads

$$\frac{\partial^2 W}{\partial R^2} + \frac{1}{R}\frac{\partial W}{\partial R} + \frac{1}{R^2}\frac{\partial^2 W}{\partial\varphi^2} = -1 \ . \tag{15}$$

The boundary conditions necessary for solution of Eq. (15) result from the symmetry of the problem and equilibrium of the free surface and are following:

$$W = 0 \quad \text{for} \quad \varphi = \pm\Phi \tag{16}$$

$$\frac{\partial W}{\partial n} = 0 \quad \text{for} \quad R = R_b(\varphi) \ . \tag{17}$$

The letter condition can be reformulated in a more convenient form for numerical calculations:

$$\frac{\partial W}{\partial R} - \frac{1}{R_s}\frac{\partial R_s}{\partial \varphi}\frac{1}{R}\frac{\partial W}{\partial \varphi} = 0 \quad \text{for} \quad R = R_s(\varphi) \ . \tag{18}$$

Finally, the solution of the undertaken problem of flow of liquid in open triangular groove can be considered as the two related tasks. The first one requires of the solution of the two point boundary value problem defined by the nonlinear ordinary differential Eq. (9) with boundary conditions (10–12) and is aimed at a determination of the shape of the free surface. Then having determined the shape of the free surface, the second task consists in a solution of the boundary value problem defined by Poisson's equation (15) with boundary condition (16), satisfied on the groove's wall, and the boundary condition (18) that has to be satisfied on the free surface.

3 Solution procedure

In the earlier authors paper (Kaczmarek, Kolodziej and Musielak 1988) a numerical method for determination of the shape of free surface of stagnant liquid in triangular groove was proposed assuming variable amount of liquid, and wide range of coefficient of surface tension and of density of liquid. In terms introduced in the present paper the two-point boundary value problem defined by Eq. (9) and boundary conditions (10–12) was considered assuming a wide distribution of the Bond number and lack of liquid flow. In the present paper the results of (Kaczmarek, Kolodziej and Musielak 1988) are utilized in order to solve the problem of flow governed by (15) with boundary conditions (16) and (18). The results consist of a discrete distribution of two coordinates of the free surface: radius R versus angle $\varphi(I)$, $R_s(\varphi(I)) = R(I)$, and distribution of the derivative of the radius R with respect to the angle $\varphi : \mathrm{d}R_s(\varphi(I))/\mathrm{d}\varphi = DR(I)$, where $I = 1, 2, \ldots, M$, and M is a chosen number of points on the free surface.

The analytical solution which exactly satisfies Eq. (15) and boundary condition (16) is assumed in one of the two alternative forms:

1. If the basal angle $\Phi = 45°$, then

$$W(R,\varphi) = \frac{-R^2}{4}\left[1 - \frac{4}{\pi}(\varphi \sin 2\varphi - \cos 2\varphi \ln R)\right] + \sum_{m=1}^{\infty} B_m R^{4m-2} \cos[(4m-2)\varphi] \tag{19}$$

2. and for other basal angles

$$W(R,\varphi) = \frac{-R^2}{4}\left(1 - \frac{\cos 2\varphi}{\cos 2\Phi}\right) + \sum_{m=1}^{\infty} B_m R^{\frac{(2m-1)\pi}{2\Phi}} \cos\left[\frac{(2m-1)\pi\varphi}{2\Phi}\right] \tag{20}$$

where B_m are unknown constants.

Considering the first N terms of the solution (19) or (20) and requiring that the boundary condition (18) is satisfied exactly in M points of the free surface one obtains a linear system of equations

$$\sum_{J=1}^{N} A(I,J)B(J) = C(I) \quad I = 1,2,3,\ldots,M \tag{21}$$

where for $\Phi = 45°$

$$\begin{aligned} C(I) = \tfrac{1}{2}R(I)\{1 - 4[&\varphi(I)\sin(2\varphi(I)) \\ &- \cos(2\varphi(I))\ln(R(I))]/\pi\} \\ &+ R(I)\cos(2\varphi(I))/\pi \\ &+ [\sin(2\varphi(I)) + 2\varphi(I)\cos(2\varphi(I)) \\ &+ 2\sin(2\varphi(I))\ln(R(I))]DR(I)/\pi \end{aligned} \tag{22}$$

and

$$\begin{aligned} A(I,J) = (4J-2)\{&R(I)^{4J-3}[\cos((4J-2)\varphi(I)) \\ &+ DR(I)\sin((4J-2)\varphi(I))/R(I)]\} \end{aligned} \tag{23}$$

and for other basal angles Φ

$$C(I) = \frac{1}{2}R(I)\left[1 - \frac{\cos(2\varphi(I))}{\cos(2\Phi)}\right] - \sin(2\varphi(I))DR(I)/(2\cos(2\Phi)) \tag{24}$$

and

$$A(I,J) = \frac{(2J-1)\pi}{2\Phi} R(I)^{\frac{(2J-1)\pi}{2\Phi}-1}\left\{\cos\left[\frac{(2J-1)\pi}{2\Phi}\varphi(I)\right] + \frac{DR(I)}{R(I)}\sin\left[\frac{(2J-1)\pi}{2\Phi}\varphi(I)\right]\right\} \tag{25}$$

If $N < M$ the system of equations (21) constitutes an overdetermined system of linear algebraic equations which can be solved by the method of least square, leading to the system of N equations with N unknowns. In the matrix notation the system has the following form

$$(\mathbf{A}^{\mathrm{T}}\mathbf{A})\mathbf{B} = \mathbf{A}^{\mathrm{T}}\mathbf{C} \tag{26}$$

where $\mathbf{A}^{\mathrm{T}}$ is the transpose of the matrix defined by (23) and (25).

The solution of (26) using the Gauss elimination method leads to the distribution coefficients which are present in solution (19) or (20).

While the practical problems of flow in triangular grooves are considered the essential quantities which are analyzed are the average velocity of liquid over the groove crossection and the friction factor. The dimensional average velocity of liquid in the groove is defined as

$$\bar{W} = \frac{1}{\tan \Phi}\int_{-\Phi}^{\Phi}\int_{0}^{R_s(\varphi)} RW(R,\varphi)\mathrm{d}R\,\mathrm{d}\varphi \ . \tag{27}$$

In turn the hydraulic diameter being necessary to determine the Reynolds number is a ratio of the surface area of liquid cross-section and of the wetted radius,

$$D_h = \frac{2\tan \Phi}{R_s(\Phi)} \ . \tag{28}$$

The friction factor is then expressed as

$$f = \frac{K}{\mathrm{Re}} \tag{29}$$

where the Reynolds number reads

$$Re = \bar{W} D_h \left(\frac{g \sin \beta}{\nu} b^2 \right) \qquad (30)$$

and the dimensionless coefficient K determining the friction factor is

$$K = \frac{2 D_h^2}{\bar{W}} \qquad (31)$$

Using the solution of the boundary value problem for the flow of liquid gives necessary values of local velocity which can be used to calculate the average liquid velocity. The numerical integration is realized by division of the region occupied by liquid into triangular elements. Each triangle is tied to the edge of the groove and to the free surface (the other two edges). In each triangular element seven Gauss points are located for integration.

4 Discussion of the results

One should notice that within the presented considerations the problem of determination of the shape of the free surface is solved separately from the problem of flow. In the first step for the given Bond number, basal angle of the groove, and contact angle the shape of the free surface is determined. Then, the laminar flow along the triangular groove for the known boundary of fluid is considered. Such approach is possible due to the relatively simple geometry of the problem and assumption that the flow is laminar and fully developed. For more complex flow geometry as for example considered in the papers by Nakayama and Yazaki 1993, Yamaguchi 1996, Nakayama and Shibata 1998 the problem of determination of the shape of free surface taking into account the effect of interfacial tension and the flow problem must be solved simultaneously.

In the classical paper by Ayyaswamy, Catton and Edwards 1974 similar to the above considered problem was solved yet there are two essential differences between the proposed approaches. First, the former approach assumes a constant curvature of the free surface instead of calculation. The solutions of the flow problem were achieved by using boundary Galerkin method or boundary collocation method, respectively. Although the form of the assumed analytical solutions is identical in both cases (except for $\Phi = 45^\circ$ which was not considered by Ayyaswamy, Catton and Edwards 1974) the calculation of the matrix of the system and of the vector of free terms takes much more time in the Galerkin method because of integration involved in determination of every component of the system. There is no such time consuming integration in the boundary collocation method.

The solution of the gravitational flow in triangular grooves was obtained through the eigenfunction expansion and boundary collocation method in the least square sense. It means that the governing Eq. (15) and boundary condition (16) are fulfilled exactly leading to eigenfunction expansion given by (19) or (20). The approximate character of the solution results only from the restriction of the numbers of terms in the infinite series, which constitute the solution up to N first terms. Then it is required that the solution has to satisfy the boundary condition (18) in a finite number of points, M. One has to notice, however, that the calculated distribution coefficients, B_m, decrease fast with increasing m, what is illustrated in Table 1. It is also worth noticing that the method of eigenfunction expansion and boundary collocation is very useful to solve the undertaken problem, because like in many other cases for mechanical problems (see Kolodziej 1987 for a review) the method, when applicable, is quite efficient and yields accurate results. In comparison with the other very popular numerical methods such as finite element or finite difference methods it is known that the latter methods require far more computational steps (about the square) than the boundary collocation method. In addition, in the boundary collocation method the solution is given by explicit formula, which allows to calculate easily the maximum error of satisfaction of the boundary condition between collocation points. Of course, direct numerical integration (FEM, FDM) is more versatile in terms of complex geometry and nonlinear problems.

Table 1. Examples of coefficients of expansion in solution (20)

	Bo = 2.00 $\Phi = 70$ $\Theta = 45$	Bo = 5.00 $\Phi = 55$ $\Phi = 45$
1	9.24064153737965E-0001	1.18400858995588E+0000
2	7.84496544129952E-0003	7.36777158499981E-0003
3	1.04667811381537E-0004	−4.50108207736381E-0005
4	1.74703361762169E-0005	1.38618805054487E-0005
5	2.57126462243500E-0006	7.58937229060091E-0007
6	9.51692756749238E-0007	1.72094948116754E-0006
7	2.31613088928680E-0007	9.35626343643216E-0007
8	3.43499643619270E-0008	3.05448292654265E-0007
9	2.72671521811664E-0009	5.04945735619946E-0008
10	9.11842793759512E-0011	3.46706412956174E-0009

Bearing in mind the practical importance of the problem considered in this paper the main purpose of this section is to present and discuss numerical results derived for friction factor determined for flow in triangular grooves. The knowledge of this coefficient is fundamental one in problems involving the prediction of heat transfer coefficients (Edwards 1973) processes of intensive evaporation (Ha and Peterson 1994) and other phenomena related to gravitational flow in triangular grooves when the effects of surface tension are essential. As the reference the results published in the well known paper (Ayyaswamy, Catton and Edwards 1974) are used where the friction factor for capillary flow in triangular grooves is calculated for the case of a very low Bond number. Then, the assumption of the constant radii of curvature of the free surface determined by the contact and the basal angles is adopted. It is however evident that the constant radii approximation of the curvature gives relatively good prediction if compared with the solution treated as the classical open channel flow problem when the liquid surface is flat and the surface tension is not taken into account (Straub, Silberman and Nelson 1958).

In order to show the influence of the finite Bond number equivalently of the curvature of the free surface on friction factor the comparison of the latter parameter as function of the Bond number is given in Fig. 2. On the

Table 2. Tabulated values of dimensionless friction factor K versus Bond number Bo, groove angle Φ and contact angle Θ

Φ	Θ	K					
		Ayyaswamy et al. 1974	Bo = 5.0	Bo = 1.0	Bo = 0.1	Bo = 0.01	Bo = 0.001
70	45		54.92855	56.30342	56.57066	56.59574	56.59813
65	45		54.80490	55.60718	55.77903	55.79633	55.79793
60	45		54.43993	54.88092	54.97956	54.99024	54.99119
55	45		53.90546	54.11651	54.16464	54.17054	54.17101
50	45		53.23921	53.31225	53.32905	53.33184	53.33200
45	45		52.46960	52.46959	52.46960	52.47065	52.47065
40	45	55.770	51.62138	51.59195	51.58517	51.58544	51.58537
35	45		50.71714	50.68452	50.67703	50.67710	50.67702
30	45	53.508	49.77787	49.75475	49.74946	49.74960	49.74954
25	45		48.82372	48.81299	48.81056	48.81083	48.81080
20	45	51.277	47.87388	47.87234	47.87201	47.87234	47.87234
15	45		46.94058	46.94265	46.94313	46.94342	46.94343
10	45	49.260	46.00358	46.00476	46.00502	46.00521	46.00522
5	45	48.471	45.00712	45.00673	45.00664	45.00670	45.00670
70	40		53.11795	54.42273	54.69852	54.72482	54.72730
65	40		53.10805	53.82784	53.99786	54.01547	54.01713
60	40		52.85891	53.21675	53.30604	53.31594	53.31683
55	40		52.44624	52.57817	52.61197	52.61619	52.61654
50	40	56.720	51.90771	51.90764	51.90761	51.90823	51.90824
45	40		51.27027	51.20449	51.18741	51.18613	51.18595
40	40	54.703	50.55709	50.47052	50.44811	50.44614	50.44589
35	40		49.78938	49.70952	49.68899	49.68710	49.68689
30	40	52.706	48.98711	48.92755	48.91240	48.91097	48.91082
25	40		48.16970	48.13342	48.12431	48.12344	48.12334
20	40	50.758	47.35536	47.33819	47.33395	47.33353	47.33348
15	40		46.55455	46.54878	46.54737	46.54721	46.54719
10	40	49.017	45.74323	45.74152	45.74112	45.74104	45.74104
5	40	48.356	44.85935	44.85841	44.85820	44.85815	44.85815
70	35		51.20560	52.38459	52.66883	52.69620	52.69885
65	35		51.31592	51.89829	52.05389	52.07012	52.07172
60	35		51.18557	51.40559	51.46747	51.47424	51.47488
55	35		50.89510	50.89492	50.89486	50.89501	50.89501
50	35		50.48320	50.36048	50.32503	50.32130	50.32091
45	35		49.97638	49.80036	49.74985	49.74444	49.74390
40	35	53.476	49.39708	49.21524	49.16381	49.15828	49.15772
35	35		48.76597	48.60793	48.56415	48.55944	48.55898
30	35	51.772	48.10286	47.98345	47.95123	47.94776	47.94743
25	35		47.42720	47.34929	47.32891	47.32670	47.32649
20	35	50.141	46.75707	46.71462	46.70389	46.70270	46.70259
15	35		46.10174	46.08342	46.07896	46.07842	46.07837
10	35	48.719	45.43380	45.42759	45.42614	45.42591	45.42590
5	35	46.295	44.68583	44.68410	44.68371	44.68363	44.68363
70	30		49.18361	50.13998	50.41429	50.44219	50.44497
65	30		49.41882	49.77555	49.88615	49.89825	49.89945
60	30	55.315	49.40926	49.40891	49.40877	49.40884	49.40883
55	30		49.24081	49.03169	48.96382	48.95626	48.95550
50	30	53.563	48.95403	48.63861	48.53705	48.52575	48.52461
45	30		48.57593	48.22758	48.11761	48.10547	48.10426
40	30	52.058	48.12905	47.79895	47.69769	47.68664	47.68553
35	30		47.63439	47.35509	47.27246	47.26355	47.26264
30	30	50.681	47.11247	46.90042	46.84026	46.83386	46.83322
25	30		46.58376	46.44145	46.40290	46.39884	46.39845
20	30	49.407	46.06721	45.98566	45.96462	45.96244	45.96222
15	30		45.57187	45.53422	45.52496	45.52400	45.52391
10	30	48.354	45.06760	45.05458	45.05153	45.05121	45.05118
5	30	48.031	44.48245	44.47959	44.47895	44.47888	44.47887
70	25		47.04847	47.63248	47.83895	47.86232	47.86467
65	25		47.41040	47.40981	47.40956	47.40977	47.40977
55	25		47.47426	46.94735	46.75208	46.72923	46.72688
50	25	51.599	47.31031	46.70397	46.48476	46.45939	46.45680
45	25		47.05822	46.45079	46.23894	46.21486	46.21242
40	25	50.400	46.74145	46.18881	46.00453	45.98399	45.98190
35	25		46.38213	45.92074	45.77468	45.75873	45.75711

Table 2. (Continuation)

Φ	Θ	*K*					
		Ayyaswamy et al. 1974	Bo = 5.0	Bo = 1.0	Bo = 0.1	Bo = 0.01	Bo = 0.001
30	25	49.395	46.00249	45.65097	45.54594	45.53469	45.53355
25	25		45.62520	45.38554	45.31820	45.31113	45.31042
20	25	48.532	45.27155	45.13067	45.09349	45.08966	45.08928
15	25		44.95195	44.88492	44.86826	44.86659	44.86643
10	25	47.912	44.63467	44.61142	44.60592	44.60542	44.60537
5	25	47.810	44.24382	44.23935	44.23833	44.23824	44.23823
70	20		44.80317	44.80227	44.80172	44.80220	44.80220
65	20		45.28956	44.74668	44.50222	44.47074	44.46742
60	20	47.364	45.51988	44.67402	44.29897	44.25090	44.24589
55	20		45.58972	44.59494	44.16769	44.11404	44.10850
50	20	49.262	45.54455	44.51218	44.08812	44.03629	44.03096
45	20		45.41410	44.42786	44.04426	43.99870	43.99405
40	20	48.432	45.22328	44.34490	44.02396	43.98698	43.98320
35	20		44.99619	44.26732	44.01882	43.99098	43.98815
30	20	47.864	44.75787	44.20038	44.02390	44.00466	44.00272
25	20		44.53479	44.15038	44.03770	44.02572	44.02452
20	20	47.483	44.35267	44.12284	44.06045	44.05398	44.05333
15	20		44.22553	44.11417	44.08604	44.08322	44.08293
10	20	47.376	44.12197	44.08331	44.07411	44.07325	44.07316
5	20	47.539	43.96281	43.95598	43.95442	43.95425	43.95423
70	15		42.46034	41.59765	41.09556	41.01751	41.00911
65	15		43.06314	41.73527	40.99036	40.87811	40.86612
60	15	47.364	43.40573	41.83608	40.99492	40.87352	40.86070
55	15		43.58659	41.92598	41.08519	40.96944	40.95730
50	15	46.415	43.65330	42.01563	41.23977	41.13784	41.12726
45	15		43.63725	42.11204	41.44137	41.35715	41.34849
40	15	46.058	43.56528	42.22150	41.67687	41.61133	41.60464
35	15		43.46424	42.35070	41.93711	41.88927	41.88440
30	15	46.025	43.36334	42.50685	42.21662	42.18423	42.18095
25	15		43.29469	42.69749	42.51330	42.49339	42.49138
20	15	46.220	43.29092	42.92836	42.82649	42.81577	42.81469
15	15		43.37287	43.19439	43.14842	43.14371	43.14323
10	15	46.724	43.51276	43.45073	43.43587	43.43442	43.43427
5	15	47.206	43.62998	43.61971	43.61736	43.61704	43.61702
70	10		40.04693	38.00019	36.43526	36.11765	36.08192
65	10		40.75018	38.34783	36.64114	36.32550	36.29073
60	10	43.214	41.19266	38.63444	36.94941	36.66412	36.63328
55	10		41.47326	38.90269	37.34282	37.09948	37.07360
50	10	42.863	41.64073	39.17304	37.80278	37.60446	37.58363
45	10		41.72794	39.45911	38.31365	38.15875	38.14268
40	10	43.149	41.76392	39.77224	38.86356	38.74797	38.73609
35	10		41.77905	40.12323	39.44437	39.36264	39.35429
30	10	43.790	41.80772	40.52279	40.05165	39.99760	39.99211
25	10		41.88934	40.98110	40.68415	40.65147	40.64817
20	10	44.686	42.06609	41.50544	41.34184	41.32442	41.32266
15	10		42.37052	42.09045	42.01693	42.00920	42.00844
10	10	45.927	42.78565	42.68866	42.66531	42.66248	42.66224
5	10	46.798	43.23374	43.21864	43.21519	43.21366	43.21362
70	5		37.61415	34.07323	30.52614	29.52694	29.40525
65	5		38.39105	34.61997	31.21300	30.39264	30.29681
60	5	37.615	38.91271	35.08524	31.96143	31.30671	31.23346
55	5		39.27514	35.52587	32.77088	32.25993	32.20449
50	5	38.346	39.52666	35.97218	33.63232	33.24257	33.20137
45	5		39.70098	36.44546	34.53650	34.24696	34.21697
40	5	39.544	39.82920	36.96371	35.47633	35.26817	35.24698
35	5		39.94531	37.54380	36.44762	36.30404	36.28967
30	5	41.056	40.08917	38.20197	37.44922	37.35533	37.34613
25	5		40.30840	38.95323	38.48292	38.42553	38.42010
20	5	42.814	40.65843	39.80838	39.55143	39.51899	39.51615
15	5		41.19234	40.76376	40.64964	40.63208	40.63085
10	5	44.951	41.91728	41.77095	41.73569	41.72426	41.72388
5	5	46.295	42.75768	42.74156	42.73667	42.72753	42.72748

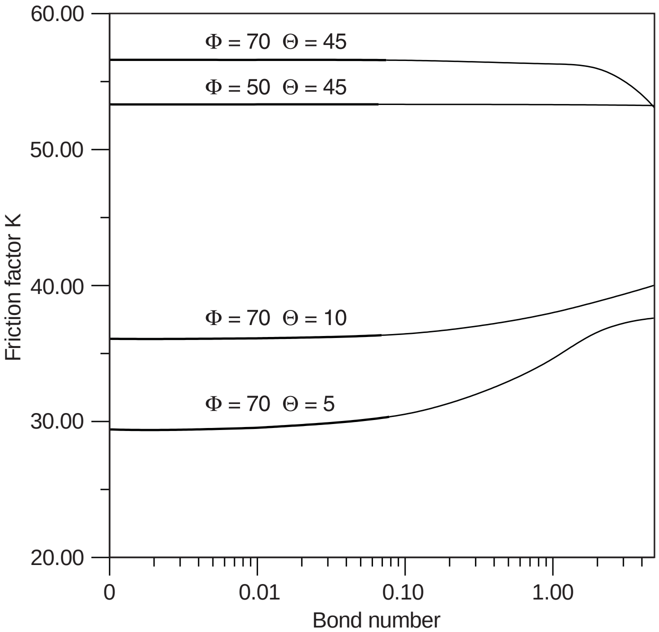

Fig. 2. Dependence of dimensionless friction factor K on Bond number Bo for various groove half angle Φ and contact angle Θ

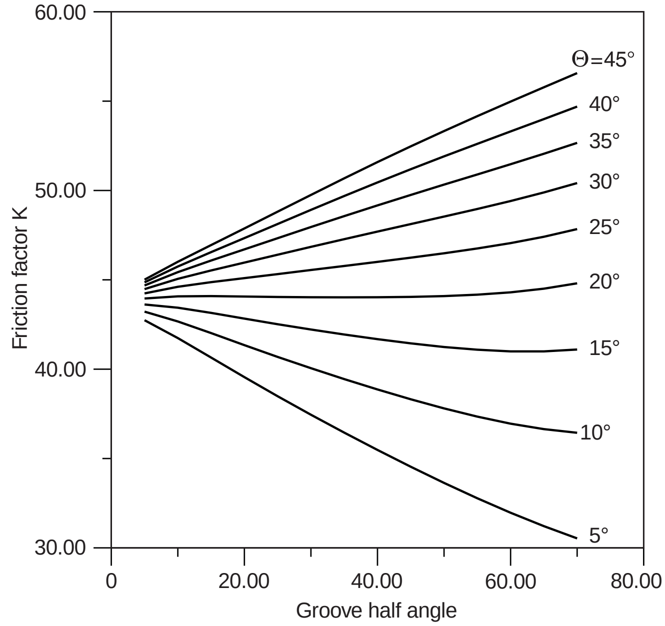

Fig. 4. Dependence of dimensionless friction factor K on the groove half angle Φ for various contact angle from $\Theta = 45$ (up curve) to 5 (down curve) with 5 deg step for Bond number Bo $= 0.1$

other hand the obtained results show that for large contact angles the dimensionless coefficient K is practically independent of Bond number (see Table 2). The dimensionless coefficient K changes significantly with Bond number for small contact angles and large basal angle (see Fig. 2). According to the method of Ayyaswamy et al. 1974 the minimal values of the position vector of free surface is fixed when the half groove angle Φ and contact angle Θ is fixed. These minimal values of the position vector as a function of Bond number on the base of proposed method is shown in Fig. 3. Generally this position vector is practically independent of Bond number for low its value but changes significantly for moderate Bond number.

Comparing the results for K from the present paper and the paper by Ayyaswamy et al. 1974 (see Table 2) one should notice that two different characteristic length were used to define all the dimensionless quantities. While in the former paper the distance from the free surface to the bottom of the groove when there is no capillary forces was chosen, as the characteristic length in the latter paper is

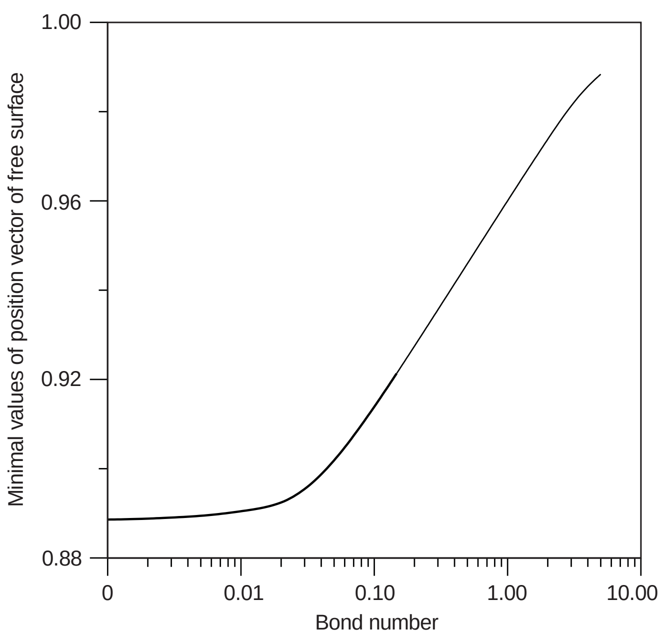

Fig. 3. Dependence of minimal values of position vector of free surface $R_s(0)$ on Bond number Bo for groove half angle $\Phi = 70^{\circ}$ and contact angle $\Theta = 5^{\circ}$

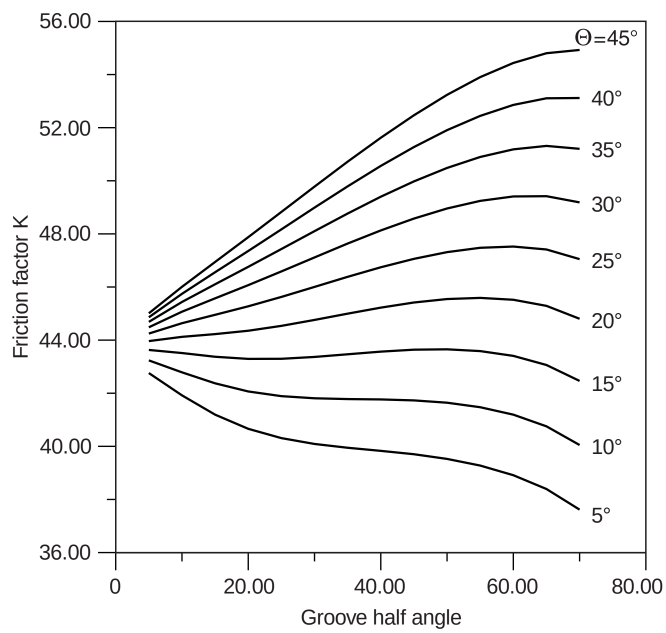

Fig. 5. Dependence of dimensionless friction factor K on the groove half angle Φ for various contact angle from $\Theta = 45$ (up curve) to 5 (down curve) with 5 deg step for Bond number Bo $= 5$

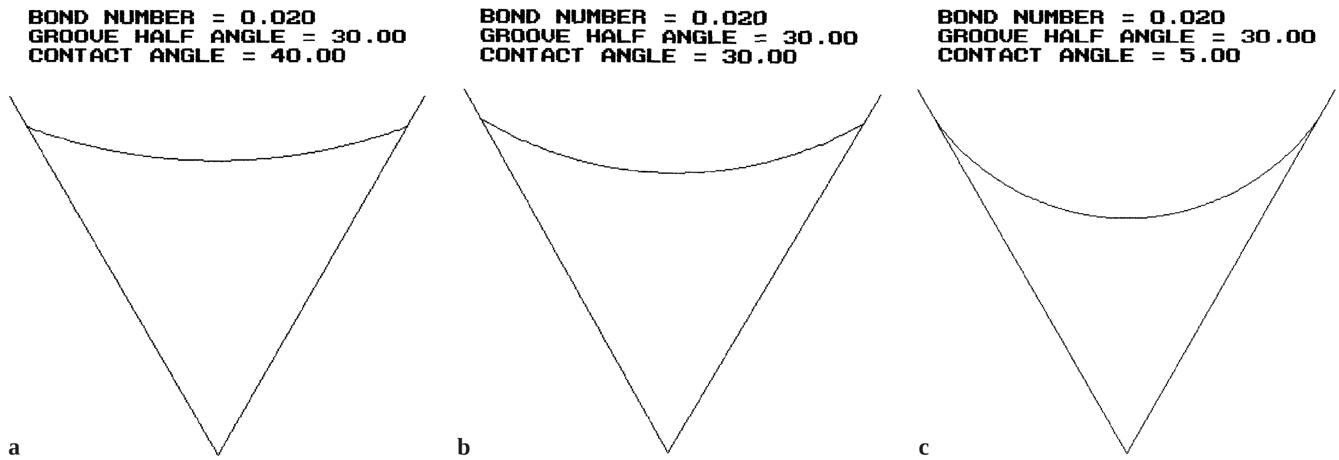

Fig. 6a–c. Shapes of free surface for chosen Bond numbers and different contact angles

equal to the groove boundary length. The main reason of such difference is related to the methods of determination of the shape of free surface.

To present quantitative results for the dependence of dimensionless friction factor on the other two parameters: the groove half angle, and contact angle the numerical results are gathered in Figs. 4–5. One can observe that dimensionless friction factor is decreasing for low values of contact angle or increasing for high values of contact angle with increasing of grove angle. Examples of shapes of free surface for chosen Bond number are given in Fig. 6.

The obtained results can be used as benchmark data useful in design of the engineering problems involving flow in triangular grooves and especially in optimizing the processes.

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