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Optimization of the Effective Thermal Conductivity of a Composite

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1. Introduction

Composite materials by definition are a combination of two or more materials. Although the idea of combining two or more components to produce materials with controlled properties has been known and used from time immemorial, modern composites were developed only several decades ago and have found by now intensive application in different fields of engineering (Vasiliev&Morozov, 2001).
These materials are used in various design to improve the characteristic of various construction and reduce their weight. The properties of these materials and the problems of obtaining structural elements based upon them have been studied by researchers and engineers all over the world. The fields of composite applications are diversified (Freger et al., 2004). They include structural elements of flying vehicles, their casinos, wings, fuselages, tails and nose cones, jet engine stators, panels form various purposes, main rotors of helicopters, heat – proofing components, construction elements such as panels, racks, shields, banking elements, etc.
Any property of a composite which is made of two (or more) materials has the value which is the resultant of a few factors. Obviously, the most important are the values of a certain property of each constituent material. However, one of the factor that also influences the resultant value of a property of a composite as a whole is its geometrical structure. Such resultant properties are commonly called effective properties of a composite. Temperature is the most important of all environmental factors affecting the behaviour of composite materials, mainly because composites are rather sensitive to temperature and have relatively low effective thermal conductivity. For instance, advanced composites for engineering applications are characterized with low density providing high specific strength and stiffness, low thermal conductivity resulting in high heat insulation, and negative thermal expansion coefficient allowing us to construct hybrid composite elements that do not change their dimensions under heating (Vasiliev & Morozov, 2001).
Because experimental evaluation of effective properties (e.g. thermal conductivity) of composites is expensive and time consuming, computational methods have been found to provide efficient alternatives for predicting the best parameters of composites, especially those having complex geometries. To achieve a reliable prediction, one needs to work on two aspects: a good description of the structural details of fibrous materials, and an efficient numerical method for the solution of energy equations through the fibrous structures (Wang et al. 2009). The need to determine the thermal conductivity of fibres for design purposes has been the motivation of work (Al-Sulaiman et al., 2006). Authors developed four

empirical formulas to predict the thermal conductivities of fibre reinforced composite laminates and their constituents. In the paper (Boguszewski et al., 2008) the analysis of structure and features of phases composite was considered in order to study heat transfer phenomena. Models of three phases composite matrix, filler and interface with discontinuities were analyzed. Distance between particles was also considered. The paper (Kidalov & Shakhov, 2009) presents results in studying the possibility of developing composites in diamond-containing systems with a view of obtaining materials with a high thermal conductivity. The main objectives of project (Weber, 2001) were to develop a model to predict thermal conductivity of the carbon filled polymer composites and to determine if synergism between the fillers exists. Thermally conductive polymer composites can replace metals in many applications. The article (Zhou&Li, 2008) presents a numerical procedure to design two-phase periodic microstructural composites with tailored thermal conductivities, which is generalized as a topology optimization problem. The objective function is formulated in a least-square of the difference between the target and effective conductivities. Various microstructures both in 2- and 3-dimensions are presented to demonstrate such a systematic procedure of conductive material design. The effective thermal conductivity enhancement of carbon fibre composites was investigated in contribution (Wang et al., 2009) using a three-dimensional numerical method. The authors of the paper (Wang&Pan, 2008) have developed a random generation-growth method to reproduce the microstructures of open-cell foam materials via computer modelling, and then solve the energy transport equations through the complex structure by using a high-efficiency lattice Boltzmann method. The effective thermal conductivities of open-cell foam materials are thus numerically calculated and the predictions are compared with the existing experimental data. In the paper (Karkri, 2010) thermal properties of composites are investigated numerically and experimentally. In the numerical study, finite elements method is used for modelling heat transfer and to calculate the effective thermal conductivity of the composite for three elementary cells, such as simple cubic, body centered cubic and face centered cubic. The effect of the filler concentrations, the ratio of thermal conductivities of filler to matrix material and the Kapitza resistance of the contact inclusion/matrix on the effective conductivity was investigated. In the paper (Brucker&Majdalani, 2005), several analytical expressions are derived for an effective thermal conductivity. These explicit solutions embody many possible heat pathways and base plate geometries that arise in microelectronic packages. From a physical stand point, the effective thermal conductivity represents a figure-of-merit that assumes an intermediate value greater than that of the coolant, and smaller than that of the metal.

The objective of this contribution is to investigate the effective hybrid numerical method to predict effective thermal conductivity of composite material with fibres distributed in matrix phase. This method is combination of finite element method and genetic algorithm (FEM-GA). FEM-GA was used to find distribution of fibre in composite domain giving maximum, minimum or required value of effective thermal conductivity. The Algorithm is implemented in Comsol Multiphysics environment using Comsol Script language (Comsol, 2007). Comsol solver uses finite element method which today has been widely employed in solving field problems arising in modern industrial practices (Zienkiewicz & Taylor, 2000).

It is assumed that both the matrix and fibres of the considered composite are homogenous, isotropic and their thermal conductivities are constant. The fibres are cylindrical, arranged parallel, continuous with circular cross-section. The fibre diameter is relatively small in comparison to their length, thus fibres can be treated as infinitely long. Fibres can be different in size and thermal properties (thermal conductivity).

2. Fibrous composite material

In the present paper, a composite material consisting of two materials is analysed. It is a fibrous material with unidirectional fibres. The material of the matrix is homogenous and its thermal conductivity is constant. Fibres are also homogenous, however, they may differ from each other when it comes to radius or thermal conductivity.

2.1 Effective thermal conductivity

Composite materials typically consist of stiff and strong material phase, often as fibres, held together by a binder of matrix material, often an organic polymer. Matrix is soft and weak, and its direct load bearing is negligible. In order to achieve particular properties in preferred directions, continuous fibres are usually employed in structures having essentially two dimensional characteristics.

Applying the fundamental definition of thermal conductivity to a unit cell of unidirectional fibre reinforced composite with air voids, one can deduce simple empirical formula to predict the thermal conductivity of the composite material with estimated air void volume percent (Al-Sulaiman et al., 2006). The ability to accurately predict the thermal conductivity of composite has several practical applications. The most basic thermal-conductivity models (McCullough, 1985) start with the standard mixture rule

$$\lambda_{eff} = \sum_{i=1}^{n} \lambda_i V_i \tag{1}$$

and inverse mixture rule

$$\lambda_{eff} = \left(\sum_{i=1}^{n} \frac{V_i}{\lambda_i}\right)^{-1}, \tag{2}$$

where λ_{eff} is the effective thermal conductivity, λ_i, V_i - thermal conductivity and volume fraction of *i-th* composite constituents (e.g. resin, fibre, void).

The composite thermal conductivity in the filler direction is estimated by the rule of mixtures. The rule of mixtures is the weighted average of filler and matrix thermal conductivities. This model is typically used to predict the thermal conductivity of a unidirectional composite with continuous fibres. In the direction perpendicular to the fillers (through plane direction), the series model (inverse mixing rule) is used to estimate composite thermal conductivity of a unidirectional continuous fibre composite.

Another model similar to the two standard-mixing rule models is the geometric model (Ott, 1981)

$$\lambda_{eff} = \sum_{i=1}^{n} \lambda_i^{V_i} \tag{3}$$

Numerous existing relationships are obtained as special cases of above equations. Filler shapes ranging from platelet, particulate, and short-fibre, to continuous fibre are consolidated within the relationship given by McCullough (McCullough, 1985).

The effective thermal conductivity for a composite solid depends, however, on the geometry assumed for the problem. In general, to calculate the effective thermal conductivity of fibrous materials, we have to solve the energy transport equations for the temperature and heat flux fields. For a steady pure thermal conduction with no phase change, no convection and no contact thermal resistance, the equations to be solved are a series of Poisson equations subject to temperature and heat flux continuity constraints at the phase interfaces.

After the temperature field is solved, the effective thermal conductivity, λ_{eff}, can be determined

$$\lambda_{eff} = \frac{L \int q dA}{\Delta T \int dA}, \tag{4}$$

where q is the steady heat flux through the cross-section area dA between the temperature difference ΔT on a distance L. Heat flow through the unit area of the surface with normal n is linked with the temperature gradient in the n-direction by Fourier's law as

$$q = -\lambda \frac{\partial T}{\partial n} \tag{5}$$

2.2 Composite structure

The elementary cell of the considered composite is a cross-sectional square and it is perpendicular to fibres direction. Perfect contact between the matrix and the cell is assumed, heat transfer does not depend on time, and only conductive transfer is considered. Also, none of materials' properties depends on temperature, so the problem is linear and can be described by Laplace equation in each domain.

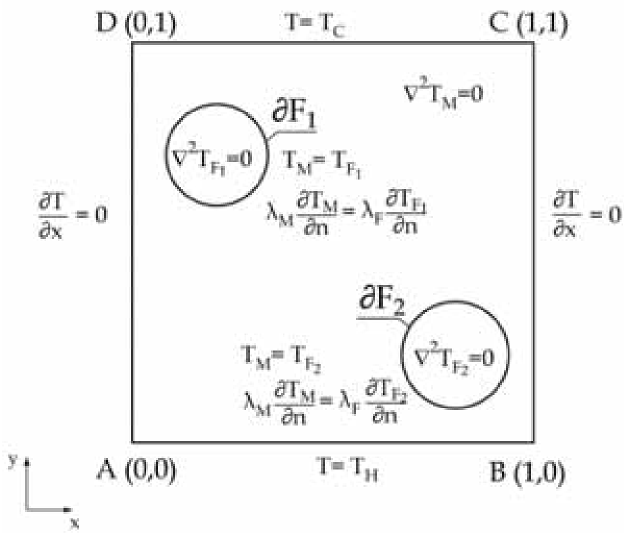

Fig. 1. Composite elementary cell structure

Governing equation of the problem both in the matrix domain and in each fibre domain takes the following form:

$$\nabla^2 \mathrm{T} = 0. \tag{6}$$

Boundary condition applied to the cell are defined as follows:

$$\frac{\partial \mathrm{T_M}}{\partial \mathrm{x}} = 0 \quad for\ \mathrm{x} = 0\ and\ \mathrm{x} = 1, \tag{7}$$

$$\mathrm{T_C} = 290\mathrm{K} \quad for \quad \mathrm{y} = 0, \tag{8}$$

$$\mathrm{T_H} = 300\mathrm{K} \quad for \quad \mathrm{y} = 1, \tag{9}$$

$$\mathrm{T_M} = \mathrm{T_F} \quad for \quad \partial\mathrm{F}, \tag{10}$$

$$\lambda_\mathrm{M} \frac{\partial \mathrm{T_M}}{\partial \mathrm{n}} = \lambda_\mathrm{F} \frac{\partial \mathrm{T_F}}{\partial \mathrm{n}} \quad for \ \partial\mathrm{F}. \tag{11}$$

Symbols used at the Fig 1. denote as follows: T_C- cooling temperature at the top of the cell, T_H- heating temperature at the bottom of the cell, λ – thermal conductivity, indices M and F refer to the matrix and fibres.
Hence, one can see that the composite is heated from the bottom and cooled from the above. Symmetry condition is applied on the sides of the cell, which means that the heat flux on these boundaries equals zero. Thermal continuity and heat flux continuity conditions are applied on the boundary of each fibre.

2.3 Relation between geometry and conductivity

As we have already mentioned, the geometrical structure of the composite material may have a great impact on the resultant effective conductivity of the composite. Commonly, researchers assume that fibres are arranged in various geometrical arrays (triangular, rectangular, hexagonal etc.) or they are distributed randomly in the cross-section. In both cases the composite can be assumed as isotropic in the cross-sectional plane. However, anisotropic materials are also very common. What is more, one may intentionally create composite because of desired resultant properties of such materials. The influence of topological configuration of fibres in unidirectional composite is shown at Figs 2A-2C. The plot (Fig 2C) shows the relation between the effective thermal conductivity and the angle β by which fibres are rotated from horizontal to vertical alignment
The minimal value of effective thermal conductivity is shown at Fig 2B, maximal value at Fig 2B[1].

3. Numerical procedures

Numerical calculations were performed by hybrid method which consisted of two procedures: finite element method used for solving differential equation and genetic algorithm for optimization. Both procedures were implemented in COMSOL Script.

3.1 Finite element method (FEM)

A case in which heat transfer can be considered to be adequately described by a two-dimensional formulation is shown in Fig 3. Two dimensional steady heat transfer in considered domain is governed by following heat transfer equation:

$$\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \dot{Q} = 0, \tag{12}$$

in the domain Ω.

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[1] All figures in this paper presenting the elementary composite cell use the same sizes and the same temperature scale as figures Fig 2A and Fig 2B, so the scales are omitted on the next figures. Isolines are presented in reversed grayscale.

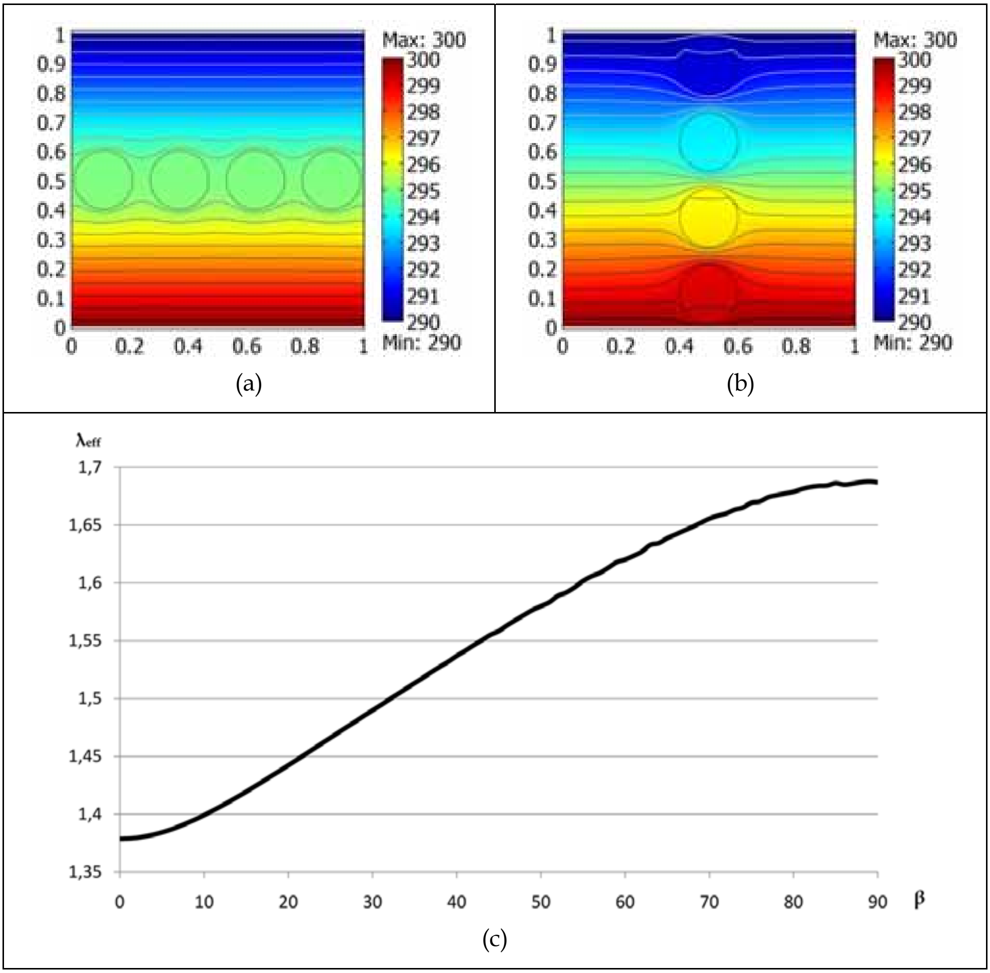

Fig. 2. (a) Horizontal alignment, λ_{eff}=1,37 (b)Vertical alignment λ_{eff}=1,68 (c) Relation between effective thermal conductivity λ_{eff} and the angle β of rotation of four fibres aligned. The conductivity of matrix λ_M=2, fibres conductivity λ_F=0.1. Fibres radius R=0.1

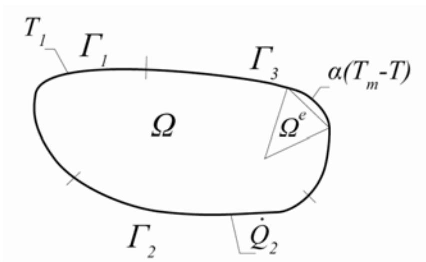

Fig. 3. Geometry of domain with boundary conditions

In the considered problem one can take under consideration three types of heat transfer boundary conditions:

$$T(x,y) = T_1 \tag{13}$$

on boundary Γ_1,

$$\left(\lambda \frac{\partial T}{\partial x} n_x\right) + \left(\lambda \frac{\partial T}{\partial y} n_y\right) = \dot{Q}_2 \tag{14}$$

on boundary Γ_2 and

$$\left(\lambda \frac{\partial T}{\partial x} n_x\right) + \left(\lambda \frac{\partial T}{\partial y} n_y\right) = \alpha(T_m - T) \tag{15}$$

on boundary Γ_3. In above equations T_m denotes external temperature, $\dot{Q}_2$ is a heat source, α – heat transfer coefficient, λ – thermal conductivity coefficient, n_x and n_y – components of normal vector to boundary.

In developing a finite element approach to two-dimensional conduction we assume a two-dimensional element having M nodes such that the temperature distribution in the element is described by

$$T^e(x,y) = \sum_{j=1}^{M} T_j^e \cdot N_j^e(x,y) = [N]\{T\} \tag{16}$$

where $N_j^e(x,y)$ is the interpolation function associated with nodal temperature T_j^e , $[N]$ is the row matrix of interpolation functions, and $\{T\}$ is the column matrix (vector) of nodal temperatures.

Applying Galerkin's finite element method (Zienkiewicz&Taylor, 2000), the residual equations corresponding to steady heat transfer equation are

$$\int_{\Omega^e} \left(\frac{\partial}{\partial x}\left(\lambda \frac{\partial T^e}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T^e}{\partial y}\right) + \dot{Q}\right) N_i^e(x,y)\,dxdy = 0. \tag{17}$$

Using Green's theorem in the plane we obtain

$$\int_{\Omega^e} \left(\frac{\partial}{\partial x}\left(\lambda \frac{\partial T^e}{\partial x} N_i^e\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T^e}{\partial y} N_i^e\right)\right) dxdy = \int_{\Gamma^e} \left(\lambda \frac{\partial T^e}{\partial x} dy - \lambda \frac{\partial T^e}{\partial y} dx\right) N_i^e \tag{18}$$

and by transforming left-hand side we obtain:

$$\begin{aligned} &\int_{\Omega^e} \left(\frac{\partial}{\partial x}\left(\lambda \frac{\partial T^e}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T^e}{\partial y}\right)\right) N_i^e\,dxdy = \\ &= -\int_{\Omega^e} \left(\lambda \frac{\partial T^e}{\partial x}\frac{\partial N_i^e}{\partial x} + \lambda \frac{\partial T^e}{\partial y}\frac{\partial N_i^e}{\partial y}\right) dxdy + \int_{\Gamma^e} \left(\lambda \frac{\partial T^e}{\partial x} dy - \lambda \frac{\partial T^e}{\partial y} dx\right) N_i^e. \end{aligned} \tag{19}$$

Using

$$\dot{Q} = \left(\lambda \frac{\partial T}{\partial x} n_x\right) + \left(\lambda \frac{\partial T}{\partial y} n_y\right) \tag{20}$$

in the Galerkin residual equation we obtain

$$\int_{\Omega^e}\left(\left(\lambda\frac{\partial T^e}{\partial x}\frac{\partial N_i^e}{\partial x}\right)+\left(\lambda\frac{\partial T^e}{\partial y}\frac{\partial N_i^e}{\partial y}\right)\right)dxdy=\int_{\Omega^e}\dot{Q}N_i^e dxdy+\int_{\Gamma^e}\left(\lambda\frac{\partial T^e}{\partial x}dy-\lambda\frac{\partial T^e}{\partial y}dx\right)N_i^e. \tag{21}$$

Taking under consideration boundary condition

$$\int_{\Omega^e}\left(\left(\lambda\frac{\partial T^e}{\partial x}\frac{\partial N_i^e}{\partial x}\right)+\left(\lambda\frac{\partial T^e}{\partial y}\frac{\partial N_i^e}{\partial y}\right)\right)dxdy= \\ =\int_{\Omega^e}\dot{Q}N_i^e dxdy+\int_{\Gamma_2^e}\dot{Q}_2N_i^e ds+\int_{\Gamma_3^e}\alpha(T_m-T_1)\,N_i^e ds, \tag{22}$$

Where

$$\dot{Q}ds=\left(\lambda\frac{\partial T}{\partial x}n_x ds\right)+\left(\lambda\frac{\partial T}{\partial y}n_y ds\right) \tag{23}$$

Using (16) in equation (22) we obtain

$$\int_{\Omega^e}\left(\left(\lambda\frac{\partial N_i^e}{\partial x}\sum_{j=1}^{M}\left(T_j^e\frac{\partial N_j^e}{\partial x}\right)\right)+\left(\lambda\frac{\partial N_i^e}{\partial y}\sum_{j=1}^{M}\left(T_j^e\frac{\partial N_j^e}{\partial y}\right)\right)\right)dxdy= \\ =\int_{\Omega^e}\dot{Q}N_i^e dxdy+\int_{\Gamma_2^e}\dot{Q}_2N_i^e ds-\int_{\Gamma_3^e}\alpha\sum_{j=1}^{M}\left(T_j^eN_j^e\right)\,N_i^e ds+\int_{\Gamma_3^e}\alpha T_m\,N_i^e ds. \tag{24}$$

The equation (24) we can rewrite for the whole considered domain which gives us the following matrix equation

$$\boldsymbol{Ka}=\boldsymbol{f} \tag{25}$$

where $\boldsymbol{K}$ is the conductance matrix, $\boldsymbol{a}$ is the solution for nodes of elements, and $\boldsymbol{f}$ is the forcing functions described in column vector.
The conductance matrix

$$\boldsymbol{K}=\boldsymbol{K}_c^e+\boldsymbol{K}_{\Gamma_3}^e \tag{26}$$

and the forcing functions

$$\boldsymbol{f}=\boldsymbol{f}_q^e+\boldsymbol{f}_{\Gamma_2}^e+\boldsymbol{f}_{\Gamma_3}^e \tag{27}$$

are described by following integrals

$$K_{c,ij}^e=\int_{\Omega^e}\left(\left(\lambda\frac{\partial N_i^e}{\partial x}\frac{\partial N_j^e}{\partial x}\right)+\left(\lambda\frac{\partial N_i^e}{\partial y}\frac{\partial N_j^e}{\partial y}\right)\right)dxdy, \tag{28}$$

$$K_{\Gamma_3,ij}^e=\int_{\Gamma_3^e}\alpha N_i^e N_j^e ds, \tag{29}$$

$$f_{q,i}^e=\int_{\Omega^e}\dot{Q}N_i^e dxdy, \tag{30}$$

$$f^e_{\Gamma_2,i} = \int_{\Gamma^e_2} \dot{Q}_2 N^e_i ds, \tag{31}$$

$$f^e_{\Gamma_3,i} = \int_{\Gamma^e_3} \alpha T_m N^e_i ds \tag{32}$$

Equations 25-32 represent the general formulation of a finite element for two-dimensional heat conduction problem. In particular these equations are valid for an arbitrary element having *M* nodes and, therefore, any order of interpolation functions. Moreover, this formulation is valid for each composite constituent.

3.2 Genetic algorithm (GA)

Genetic algorithm is one of the most popular optimization techniques (Koza, 1992). It is based on an analogy to biological mechanism of evolution and for that reason the terminology is a mixture of terms used in optimization and biology. Optimization in a simple case would be a process of finding maximum (or minimum) value of an objective function:

In GA each potential solution is called an individual whereas the space of all the feasible values of solutions is a search space. Each individual is represented in its encoded form, called a chromosome. The objective function which is the measure of quality of each chromosome in a population is called a fitness function. The optimization problem can be expressed in the following form:

$$\mathrm{f}(\hat{\mathrm{x}}) \geq \mathrm{f}(\mathrm{x}), \mathrm{x} \in \mathrm{D}, \tag{33}$$

where: $\hat{\mathrm{x}}$ denotes the best solution, f is an objective function, x represents any feasible solution and D is a search space. Chromosomes ranked with higher fitness value are more likely to survive and create offspring and the one with the highest value is taken as the best solution to the problem when the algorithm finishes its last step. The concept of GA is presented at fig 4.

Algorithm starts with initial population that is chosen randomly or prescribed by a user. As GA is an iterative procedure, subsequent steps are repeated until termination condition is satisfied. The iterative process in which new generations of chromosomes are created involves such procedures as selection, mutation and cross-over. Selection is the procedure used in order to choose the best chromosomes from each population to create the new generation. Mutation and cross-over are used to modify the chromosomes, and so to find new solutions. GA is usually used in complex problems i.e. high dimensional, multi-objective with multi connected search space etc. Hence, it is common practice that users search for one or several alternative suboptimal solutions that satisfy their requirements, rather than exact solution to the problem. In this paper GA optimizes geometrical arrangement of fibres in a composite materials as it influences effective thermal conductance of this composite. It has been developed many improvements to the original concept of GA introduced by Holland (Holland, 1975) such as floating point chromosomes, multiple point crossover and mutation, etc. However, binary encoding is still the most common method of encoding chromosomes and thus this method is used in our calculations.

3.2.1 Encoding

We consider an elementary cell of a composite that is 2-D domain and there are *N* fibres inside the cell, the position of each fibre is defined by its coordinates, which means we need

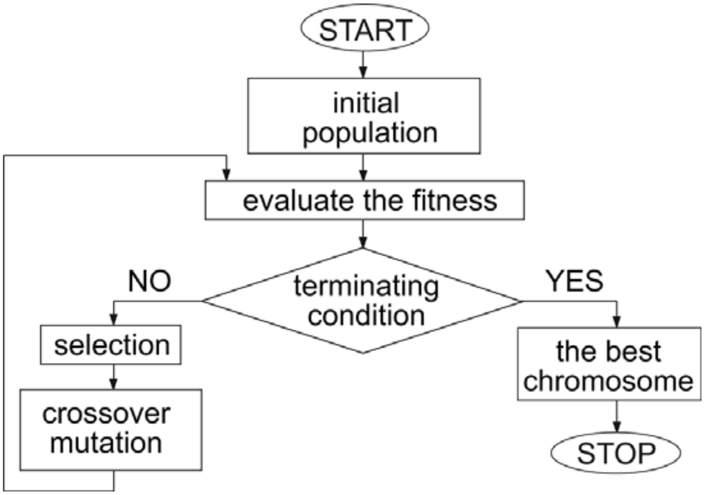

Fig. 4. Genetic algorithm scheme

to optimize *2N* variables x_i. Furthermore, it is assumed that each coordinate is determined with finite precision p_i and limited to a certain range $D_i = [a_i, b_i]$ - a, b denoting the lower and upper limit of the range respectively. It means that each domain D_i needs to be divided into $(b_i - a_i)10^{p_i}$ sub-domains. Hence we can calculate h_i – number of bits required to encode variables:

$$(b_i - a_i)10^{p_i} \le 2^{h_i} - 1. \tag{34}$$

Consequently, we can calculate the number of bits H required to encode a chromosome:

$$H = \sum_{i=1}^{2N} h_i \tag{35}$$

In our calculation we assume three significant digits precision which means we need 2^{10} bits to encode each variable.

3.2.2 Fitness and selection

Selection is a procedure in which parents for the new generation are chosen using the fitness function. There are many procedures possible to select chromosomes which will create another population. The most common are: roulette wheel selection, tournament selection, rank selection, elitists selection.

In our case, modified fitness proportionate selection also called roulette wheel selection is used. Based on values assigned to each solution by fitness function $f(x_i)$, the probability $P(x_i)$ of being selected is calculated for every individual chromosome. Consequently, the candidate solution whose fitness is low will be less likely selected as a parent whereas it is more probable for candidates with higher fitness to become a parent. The probability of selection is determined as follows:

$$P(x_i) = \frac{f(x_i)}{\sum_{k=1}^{S} f(x_k)} \tag{36}$$

where *S* is the number of chromosomes in population.

Modification of the roulette wheel selection that we introduced is caused by the fact that we needed to perform constrained optimization. The constrains are the result of the fact that fibres cannot overlap with each other. There are some possible options to handle this problem, one of which would to use penalty function. During calculations, however, it turned out that this approach is less effective than the other one based on elitist selection. We decided that in case of chromosome representing arrangement of overlapping fibres such chromosome should be replaced with the best one.

3.2.3 Genetic operators

Cross-over operation requires two chromosomes (parents) which are cut in one, randomly chosen point (locus) and since this point the binary code is swapped between the chromosomes creating two, new chromosomes, as it is shown at Fig. 5.
Mutation procedure in case of binary representation of solution is an operation of bit inversion at randomly chosen position Fig6. The following purpose of this procedure is to introduce some diversity into population and so to avoid premature convergence to local maximum.

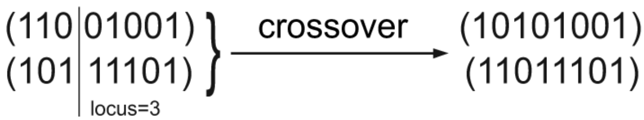

Fig. 5. Crossover procedure scheme

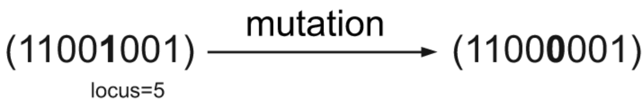

Fig. 6. Mutation procedure scheme

4. Numerical results

All optimization problems considered in this chapter are governed by Eq. 6 for each constituent of the composite with appropriate boundary conditions (7-11). In our calculations we assumed the same sizes of the unit cell i.e. 1x1cm (Fig1.). Temperatures on the lower and upper boundaries were: T_C=290K (upper), T_H=300K (lower) respectively. We analysed several cases in which the number of fibres N_f and fibres radii R were changed, also thermal conductivity of the matrix λ_M and fibres λ_F were also changed. Finite element calculation were made using second order triangular Lagrange elements. The stationary problem of heat transfer was solved using direct UMFPACK linear system solver. The mesh structure depends on the number and positions of fibres and so the number of mesh elements was not larger than 5000.
We performed three types of optimization in terms of effective thermal conductivity: minimization, maximization and determination of arrangement which gives desired value of effective thermal conductivity. In the latter case we defined the objective function as the minimization of the deviation from the expected value. The results of optimization are presented at Figs 7-9.

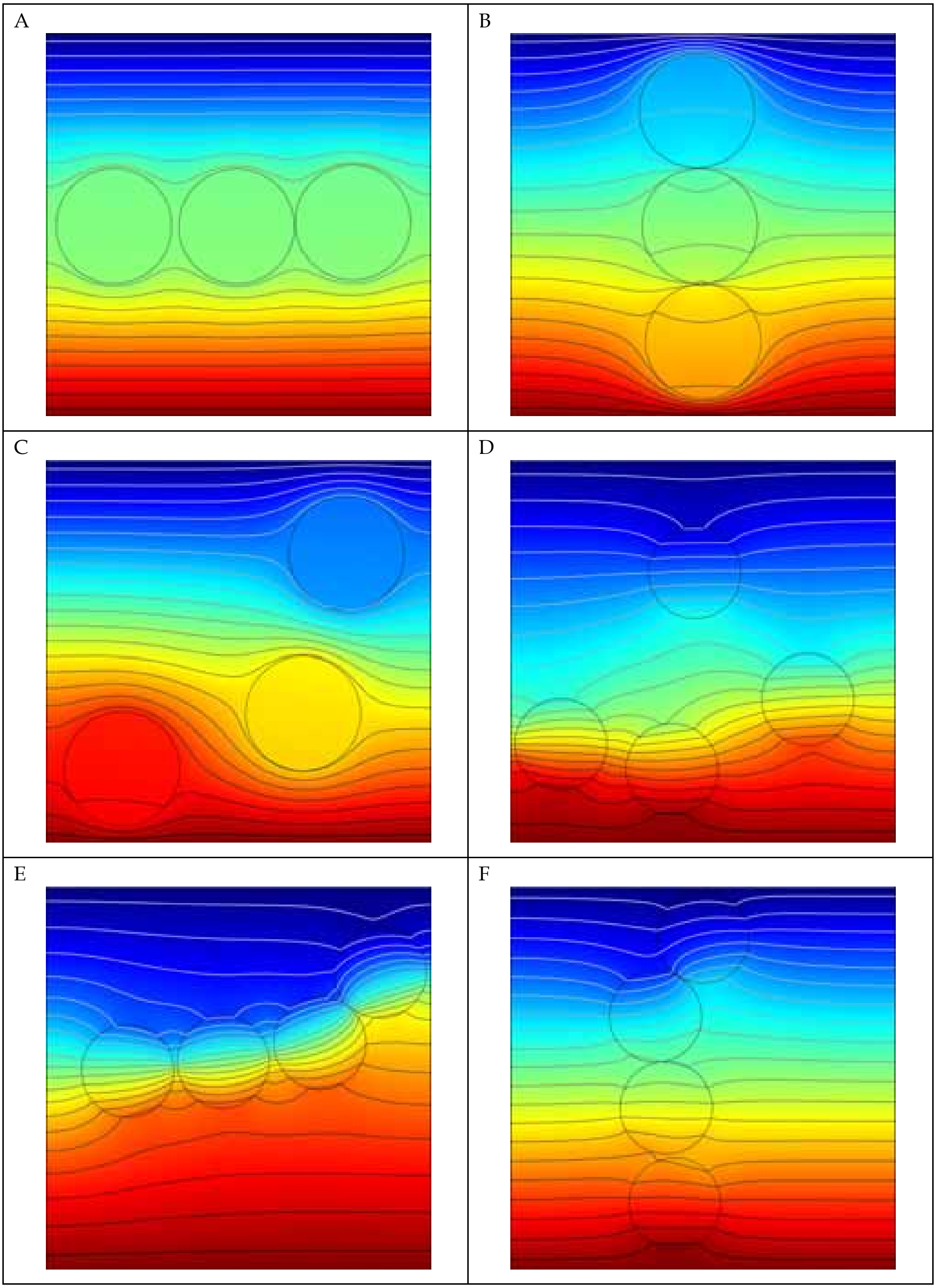

Fig. 7. Resultant arrangement for three and four fibres

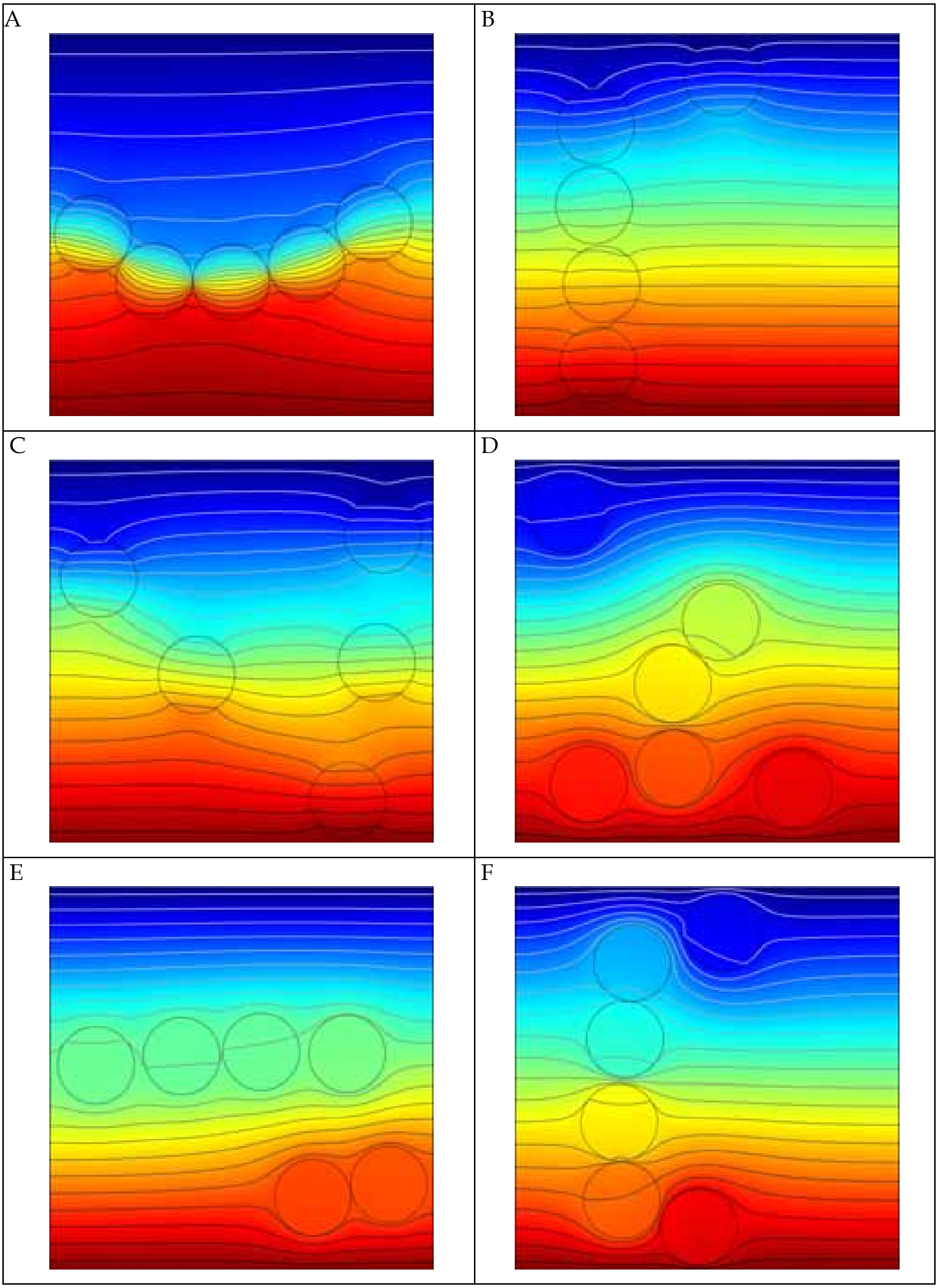

Fig. 8. Resultant arrangement for five and six fibres

4.1 Optimization of three and four fibres arrangement

In the beginning we assumed the same sizes of the fibres, as well as the same value of thermal conductivity for each fibre. Numerical values of parameters used in calculations, and the resultant effective thermal conductivity was shown in Table 1. The 'Opt.' column refers to optimization criteria i.e. minimum, maximum or expected value of λ_{eff} The column entitled $\boldsymbol{\lambda_{eff}}$ contains obtained results. Not surprisingly did minimization and maximization results agree with results presented in section 2.3. Figures 7A and 7E present the arrangement obtained during minimization. All fibres are aligned horizontally perpendicularly to heat flux direction, next to each other. In case of maximization (Figs 7B, 7F) fibres are aligned vertically – along with heat flux direction.

However, there are many possible ways of arrangement of intermediate values of effective thermal conductivity – fibres do not have to be aligned anymore as it was assumed at Fig 2C. We also presented one of possible arrangements that result in a composite with effective thermal conductivity equal to the one expected for each number of fibres: (Figs 7C, 7D). If one would like to achieve certain value of effective thermal conductivity with respect to some geometrical assumptions (for instance minimum/maximum distance between fibres) it is also possible to perform such optimization, however penalty function should be implemented or objective function modified to include such conditions.

Figure's number	N_F	R	λ_F	λ_M	Opt.	$\boldsymbol{\lambda_{eff}}$
Fig 7A	3	0.15	2.0	0.1	Min	**0.13**
Fig 7B	3	0.15	2.0	0.1	Max	**0.23**
Fig 7C	3	0.15	2.0	0.1	0.15	**0.15**
Fig 7D	4	0.12	0.1	2.0	1.35	**1.35**
Fig 7E	4	0.12	0.1	2.0	Min	**1.1**
Fig 7F	4	0.12	0.1	2.0	Max	**1.56**

Table 1. The values assigned for calculations and the resultant λ_{eff} for three and four fibres

4.2 Optimization of five and six fibres arrangement

Calculation performed for five and six fibres were similar to those presented above for three and four fibres. However, the more fibres the more complex problem. As it was mentioned in section 3.2.1 each fibre is described by two variables changing within the range [0,1] with the 10^{-3} precision which means 2^{10} bits. Consequently, by adding one fibre we enlarge the search space by 2^{20} elements. So, the search space dimension for three fibres arrangement optimization equals 2^{60}, while for six fibres it equals 2^{120}. The size of search space has a direct impact on calculation time and so it takes far more time to find optimal solution.

The terminating condition of GA was set to 2000 iterations for three and four fibres. It resulted in almost perfect arrangement in case of three fibres whereas the arrangement for four fibres was not equally well. While increasing the number of fibres to five and six fibres, we also increased the number of iteration to 10000.

Another important aspect of the considered problem was that in case of five and six fibres of assumed radii (Table 2) it was not possible to align them in one row so the relation presented in section 2.3 could not be applied anymore.

The minimization results for five and six fibres were presented at Figs 8A and 8E, the maximization results at Figs 8B and 8F and the arrangement for expected value of effective thermal conductivity at Figs. 8C, 8D. One can notice that the arrangement of fibres is also close to horizontal in case of minimization and close to vertical in case of maximization, although fibres are not localised next to each other and initialization of the second row in case of six fibres can be observed. In general, however, we may not assume that fibres are always aligned in rows in case of minimum and maximum values of effective thermal conductivity. The situation changes when the thermal conductivity of fibres is not the same in each fibre. The result for such situation was presented in the next section.

	N_F	R	λ_F	λ_M	Opt.	λ_{eff}
Fig 8A	5	0.1	0.1	2.0	Min	**1,0**
Fig 8B	5	0.1	0.1	2.0	Max	**1,61**
Fig 8C	5	0.1	0.1	2.0	1.5	**1.5**
Fig 8D	6	0.1	2.0	0.1	0.15	**0.15**
Fig 8E	6	0.1	2.0	0.1	Min	**0.13**
Fig 8F	6	0.1	2.0	0.1	Max	**0.19**

Table 2. The values assigned for calculations and the resultant λ_{eff} for five and six fibres

4.3 Optimization of four and five fibres arrangement with different radii and thermal conductivity of fibres

Apart from the simplest case in which the composite consisted of identical fibres we also analysed the case in which fibres differ from each other. We used two sizes of fibres with different values of thermal conductivities. All parameters used in calculations were presented in Table 3. The symbol N_R denotes the number of fibres having the same dimension and properties.

	N_F	N_R	R	λ_F	λ_M	Opt.	λ_{eff}
Fig 9A	4	2	0.12	0.1	2.0	Min	**1.68**
		2	0.15	10			
Fig 9B	4	2	0.12	0.1	2.0	Max	**2.39**
		2	0.15	10			
Fig 9C	4	2	0.12	0.1	2.0	2.0	**2,0**
		2	0.15	10			
Fig 9D	5	4	0.075	0.1	0.1	1.85	**1.85**
		1	0.15	10			
Fig 9E	5	4	0.075	0.1	0.1	Min	**1.65**
		1	0.15	10			
Fig 9F	5	4	0.075	20.1	0.1	Max	**2.08**
		1	0.15	10			

Table 3. The values assigned for calculations and the resultant λ_{eff} for four and five fibres of different radii and thermal conductivities

We performed the optimization of the arrangement of four and five fibres in a composite cell. The minimization results were presented at Figs 9A, 9E while maximization at Figs 9B, 9F. The arrangements obtained for the assumed values of effective thermal conductivity for four and five fibres were presented at Figs 9C,9D respectively. It is remarkable, that in these

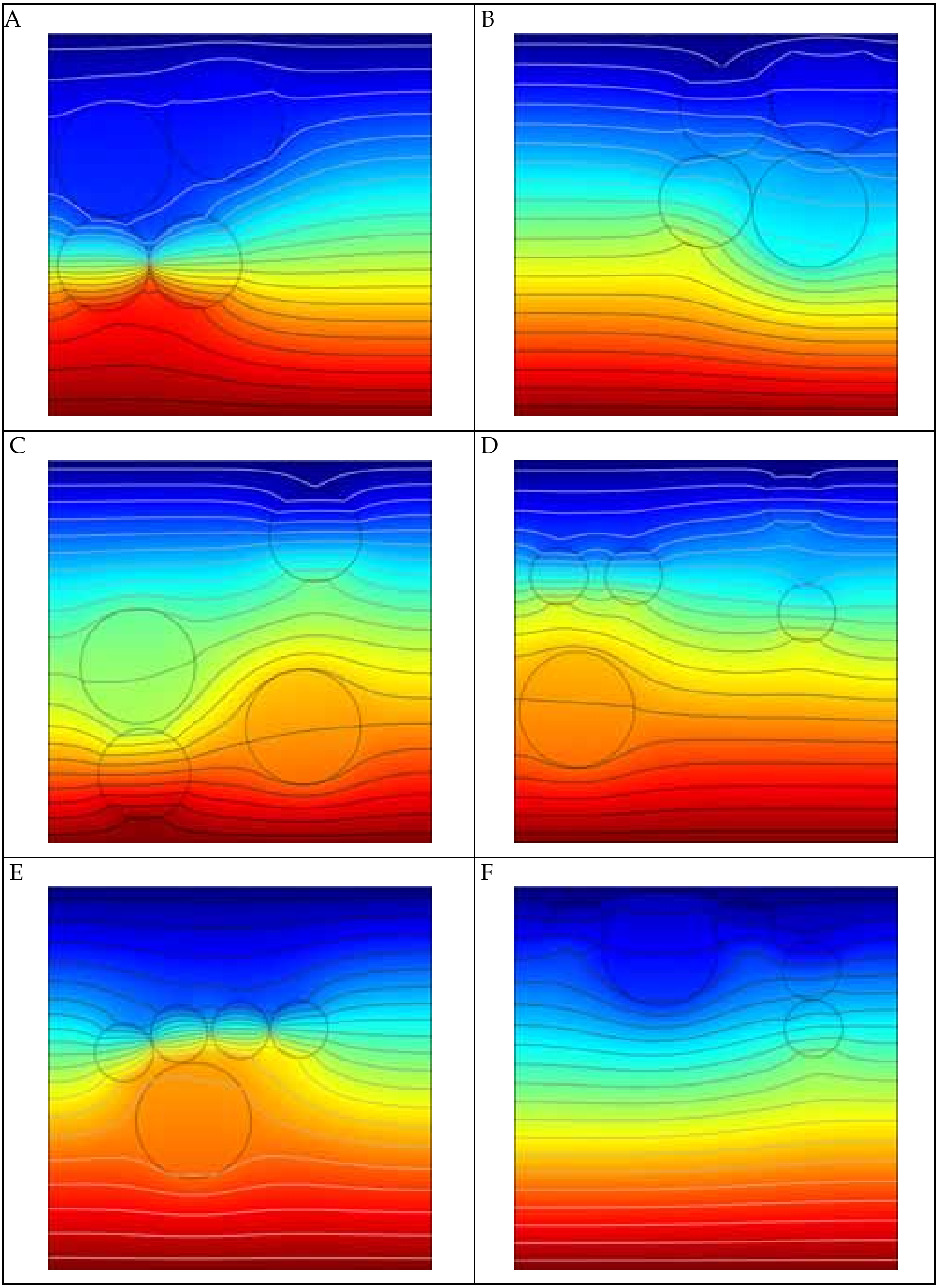

Fig. 9. Resultant arrangements for fibres of different sizes and thermal conductivities

cases the optimal arrangement of fibres is no longer that predictable. Fibres are not aligned in a row, although there was enough space. However, fibres still tend to be close to each other but spatial configuration is changed.

5. Conclusion

This study has examined the effect of multi fibres filler in composite on thermal conductivity. Three types of optimization were performed in terms of effective thermal conductivity: minimization, maximization and determination of arrangement which gives expected value of effective thermal conductivity. Hybrid method combining optimization with genetic algorithm and differential equation solver by finite element method were used to find optimal arrangement of fibres position in composite matrix was used in this work. Proposed algorithm was implemented in Comsol Multiphysics environment.

It was proved that the geometrical structure of the composite (matrix and filler arrangement) may have a great impact on the resultant effective conductivity of the composite. In many research works it is assumed that fibres are arranged in various geometrical arrays or they are distributed randomly in the cross-section.

Through this study, some areas were found that need to be investigated further. Composite constituents can be anisotropic, and with temperature dependent thermal conductivity of constituents (e.g. resin, fibre, void).

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